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A matrix big bang

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Abstract: The light-like linear dilaton background represents a particularly simple time-dependent 1/2 BPS solution of critical type-IIA superstring theory in ten dimensions. Its lift to M-theory, as well as its Einstein frame metric, are singular in the sense that the geometry is geodesically incomplete and the Riemann tensor diverges along a light-like subspace of codimension one. We study this background as a model for a big bang type singularity in string theory/M-theory. We construct the dual Matrix theory description in terms of a (1+1)-d supersymmetric Yang-Mills theory on a time-dependent world-sheet given by the Milne orbifold of (1+1)-d Minkowski space. Our model provides a framework in which the physics of the singularity appears to be under control.

Keywords: Models of Quantum Gravity, M(atrix) Theories, Superstrings and Heterotic Strings.

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1. Introduction

One of the outstanding questions facing string theory is how to describe a cosmological singularity like the big bang. For recent cosmological scenarios where the big bang singularity plays a crucial role, see for instance [1]. What prior work has taught us is that perturbative string theory breaks down on many toy model space-times that include space-like and light-like curvature singularities [2 − 6]. See [7 − 10] for some related work. To capture the physics of the singularity, a complete non-perturbative description of string theory appears to be necessary.[1] In this work, we will present a particularly clean example of a cosmological singularity that admits a holographic dual description via Matrix Theory [12] (for reviews, see for instance [13]). Prior examples of holographic descriptions, in the sense of AdS/CFT, appear in [9, 14]. See [15] for some other ideas relating Matrix theory and cosmology.

The backgrounds that we will consider are linear dilaton backgrounds which are key ingredients in some of the oldest exact solutions of string theory [16]. The dilaton, ϕ, is identified with a direction in space-time. If the direction is time-like, the solution is

[1]See, however, the very recent paper [11], which claims that certain space-like singularities are replaced by a tachyon condensate phase within perturbative string theory.

cosmological and non-supersymmetric. By definition, we lose perturbative control over these backgrounds when the string coupling

$$g_s = e^{\phi} \tag{1.1}$$

becomes large.

In this work, we want to consider a simple variant of these cosmologies where we choose light-cone coordinates in space-time and identify,

$$\phi = -QX^+ \,, \tag{1.2}$$

where Q is a constant. This kind of dilaton profile appears as an ingredient in many supergravity solutions like some plane-wave backgrounds. To construct a solution of string theory, we also need to specify a 10-dimensional space-time metric. This metric could describe some non-trivial compactification. For simplicity, we will take flat Minkowski space with coordinates $X^\mu = (X^+, X^-, X^i)$ and metric,

$$ds^2_{10} = -2dX^+dX^- + \sum_i (dX^i)^2 \tag{1.3}$$

as our 10-dimensional string metric.

This background is a remarkably simple, time-dependent string solution. Note that the parameter Q appearing in (1.2) can be scaled to any non-zero value using the boost symmetry

$$X^+ \to \alpha X^+ \,, \qquad X^- \to \alpha^{-1} X^- \,,$$

where α is non-zero. To see that flat space is still a string solution in the presence of this linear dilaton, we only need to note that a light-like linear dilaton (unlike a space-like or time-like linear dilaton) makes no contribution to the conformal anomaly.

From the perspective of string frame, the only time-dependence appears in the coupling constant. However, the corresponding Einstein frame metric is given by

$$ds^2_E = e^{QX^+/2}\, ds^2_{10} \,. \tag{1.4}$$

Viewed in Einstein frame, this space-time originates at a big bang as $X^+ \to -\infty$ since the scale factor goes to zero. In the following section, we will study this solution as a model for a big bang, and describe perturbative string quantization in this background. In section 3, we derive the Matrix description of this background which involves Matrix strings propagating on a time-dependent world-sheet. The world-sheet is described by the two-dimensional metric

$$ds^2 = e^{2\eta}\left(-d\eta^2 + dx^2\right) \tag{1.5}$$

with $x \sim x + 2\pi$. This metric describes the future quadrant of the Milne orbifold, which can be thought of as flat space with a boost identification. The curvature singularity of the metric (1.4) corresponds via Matrix theory to the Milne singularity at $\eta = -\infty$, where the x-circle shrinks to zero size. This is the "big bang". Time evolution from the big bang to the asymptotic regime corresponds in the Matrix description to renormalization group flow from the UV to the IR. The physics near the big bang is described by weakly-coupled Yang-Mills theory. In section 3.2, we argue that our Matrix description remains decoupled from gravity even at the singularity. At late times, the matrix degrees of freedom re-organize themselves into weakly-coupled strings, and a conventional space-time picture emerges.

It is worth noting that the Milne orbifold has been studied as a space-time background for closed string propagation. What can be concluded from this work is that perturbative string theory breaks down because of large gravitational backreaction from the singularity. In our case, the Milne orbifold appears as the Matrix string world-sheet. Matrix string theory should capture the non-perturbative physics of the space-time singularity.

In the final section, we mention a few of the possible generalizations of the light-like linear dilaton background.

2. The light-like linear dilaton

2.1 The light-like linear dilaton as a big bang cosmology

We begin by considering the light-like linear dilaton as a background of type IIA string theory. This defines an exact CFT that describes string propagation in flat space-time with a varying string coupling given by

$$g_s = e^{-QX^+} . \tag{2.1}$$

The space-time theory is free at late times ($X^+ \to \infty$), and strongly coupled at early times ($X^+ \to -\infty$).

This background preserves one-half of the 32 flat space supersymmetries. To see this, we need to check the supersymmetry variations of the gravitino and dilatino. Only the dilatino, λ, feels the presence of this linear dilaton background via the term in its supersymmetry variation,

$$\delta\lambda \sim \Gamma^+ \partial_+ \phi \, \epsilon , \tag{2.2}$$

where ϵ is the supersymmetry parameter. However, there are 16 solutions to the condition

$$\Gamma^+ \epsilon = 0 , \tag{2.3}$$

so half the supersymmetry is preserved. This is rather crucial since as $X^+ \to -\infty$, we have enough control from supersymmetry to determine a good strong coupling description. The spectrum in the weak coupling regime where $X^+ \to \infty$ is determined from the perturbative string quantization to be described in section 2.2.

As g_s becomes large, we expect this background to lift to a solution of M-theory. The 11-dimensional metric

$$ds^2 = e^{2QX^+/3} ds_{10}^2 + e^{-4QX^+/3} (dY)^2 , \tag{2.4}$$

with Y the eleventh direction, governs the strong coupling limit of this background. We should check that the background defined by (2.4) is not trivial. We define an orthonormal basis of 1-forms

$$e^i = e^{QX^+/3} dX^i , \qquad e^+ = e^{QX^+/3} dX^+ , \qquad e^- = e^{QX^+/3} dX^- , \qquad e^y = e^{-2QX^+/3} dY \tag{2.5}$$

with respect to which the metric takes the canonical form

$$ds_{11}^2 = -2e^+e^- + (e^i)^2 + (e^y)^2\,. \tag{2.6}$$

Up to symmetry, the corresponding spin connection has non-vanishing components

$$\omega_{i+} = \frac{Q}{3}e^{-QX^+/3}\,e^i\,, \qquad \omega_{y+} = -\frac{2Q}{3}e^{-QX^+/3}\,e^y\,, \qquad \omega_{-+} = -\frac{Q}{3}e^{-QX^+/3}e^+\,. \tag{2.7}$$

The non-vanishing curvature 2-forms are

$$\begin{aligned} R_{+i} &= \frac{Q^2}{9}e^{-2QX^+/3}e^+\wedge e^i\,,\\ R_{y+} &= \frac{8Q^2}{9}e^{-2QX^+/3}e^+\wedge e^y\,, \end{aligned} \tag{2.8}$$

with respect to the orthonormal basis (2.5). The non-vanishing components of the Riemann tensor in a coordinate basis are (up to symmetry)

$$\begin{aligned} R_{+i+i} &= \frac{Q^2}{9}e^{2QX^+/3}\,,\\ R_{+y+y} &= -\frac{8Q^2}{9}e^{-4QX^+/3}\,. \end{aligned} \tag{2.9}$$

It is easy to see from these equations that the Ricci tensor vanishes, as it should for a purely gravitational M-theory solution.

It might appear that the 11-dimensional metric (2.4) has a 'singularity' at both $X^+ \to +\infty$ and $X^+ \to -\infty$ since in both limits some metric components go to zero. The difference between these two limits is, however, that the $X^+ \to -\infty$ singularity occurs at finite geodesic distance, while the $X^+ \to \infty$ singularity is at infinite distance.

The presence of the finite distance $X^+ \to -\infty$ singularity implies that the space-time is geodesically incomplete. Namely, some geodesics terminate at finite affine parameter. This is most easily seen for the lines $X^- =$ const., $X^i =$ const., which are geodesics. The geodesic equation in this case is

$$\frac{d^2X^+}{d\lambda^2} + \Gamma^+_{++}\left(\frac{dX^+}{d\lambda}\right)^2 = 0\,, \tag{2.10}$$

where $\Gamma^+_{++} = 2Q/3$. This can be integrated to give

$$e^{\frac{2}{3}QX^+}\left(\frac{dX^+}{d\lambda}\right) = \text{const.} \tag{2.11}$$

and hence the affine parameter is (up to an affine transformation)

$$\lambda = e^{\frac{2}{3}QX^+}\,. \tag{2.12}$$

We thus find that the point $X^+ \to -\infty$ corresponds to $\lambda = 0$, and hence it has finite affine distance to all points in the interior. Note that the other 'singularity' at $X^+ \to \infty$ is indeed at infinite affine parameter, $\lambda = \infty$, so it represents an asymptotic region in which the eleventh dimension happens to curl up to zero size.

One can write the metric in terms of the affine parameter λ for $\lambda > 0$ as

$$ds^2 = -\frac{3}{Q}d\lambda dX^- + \lambda ds_8^2 + \lambda^{-2}dy^2 \,. \tag{2.13}$$

In terms of these coordinates, the non-vanishing components of the Riemann tensor are

$$\begin{aligned} R_{\lambda i \lambda i} &= \frac{1}{4\lambda}\,, \\ R_{\lambda y \lambda y} &= -\frac{2}{\lambda^4}\,, \end{aligned} \tag{2.14}$$

which clearly shows that there is a curvature singularity at $\lambda = 0$, where an inertial observer experiences divergent tidal forces.

It does not make sense to consider the metric (2.13) for $\lambda < 0$ because the signature of the eight transverse dimensions changes sign. To extend to $\lambda < 0$ in a sensible way, one might try replacing the λ in front of ds_8^2 by its absolute value $|\lambda|$. However, this extension is ad hoc without some additional input beyond general relativity about how to treat the curvature singularity. What we can conclude is that there is truly a singularity in the classical gravity description of the light-like linear dilaton background.

In fact, this same conclusion also applies to the 10-d desciption in Einstein frame: namely, there exists a singularity at finite geodesic distance. To see this, rewrite the Einstein metric (1.4) in terms of its affine parameter

$$u = e^{\frac{1}{2}QX^+} \tag{2.15}$$

and a new coordinate $v = X^-$:

$$ds_E^2 = -\frac{4}{Q}du\,dv + u\sum_i (dX^i)^2 \,. \tag{2.16}$$

Defining an orthonormal basis

$$e^i = u^{1/2}dX^i\,, \qquad e^u = \frac{2}{Q}du\,, \qquad e^v = dv\,, \tag{2.17}$$

we find the following non-vanishing components of the spin connection:

$$\omega^i{}_u = \frac{Q}{4u}e^i\,, \tag{2.18}$$

and of the curvature two form:

$$R^i{}_u = \frac{Q^2}{16u^2}\,e^i \wedge e^u. \tag{2.19}$$

In a coordinate basis, we find

$$R_{iuiu} = \frac{1}{4u}\,, \tag{2.20}$$

which is indeed singular at $u = 0$.

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In Einstein frame, the Ricci tensor is non-zero:

$$R_{uu} = \frac{2}{u^2} \,. \tag{2.21}$$

This non-vanishing Ricci tensor is supported by the dilaton, which, unlike in the string frame, has a non-zero stress-energy tensor and thus contributes to Einstein's equations. We have

$$\phi = -QX^+ = -2\log u \,, \tag{2.22}$$

and thus

$$T_{uu} = \frac{1}{2}(\partial_u \phi)^2 = \frac{2}{u^2} \,. \tag{2.23}$$

This can be interpreted as follows: the singular nature of the $R_{\lambda y \lambda y}$ component of the 11-d Riemann tensor (2.14) is transferred to the stress-energy tensor of the dilaton, and hence by Einstein's equations to the Ricci tensor (2.21).

2.2 Perturbative string theory

We now describe some of the properties of the light-like linear dilaton solution in perturbative string theory. We start with the bosonic string in D dimensions. The world-sheet fields X^μ are free, but the time-dependence of the dilaton is reflected in a modified world-sheet stress tensor,

$$T(z) = -\partial X_i \partial X^i + 2\partial X^+ \partial X^- - Q\partial^2 X^+ \,. \tag{2.24}$$

The central charge is unmodified,

$$c = D \,, \tag{2.25}$$

and Q is a free parameter.

Since the world-sheet theory is free and the string coupling is small at late times, it is not difficult to construct the physical states. As in flat space-time with constant dilaton, states are labeled by momentum p_μ and an oscillator contribution. The corresponding vertex operators have the form

$$V = e^{ip_\mu X^\mu} P_N(\partial X^\mu, \bar{\partial} X^\mu, \ldots) \,, \tag{2.26}$$

where P_N is a polynomial in derivatives of the world-sheet fields X^μ of total (left and right) scaling dimension N. We take the zero mode part of the vertex operator to be a plane wave. Physical states correspond to Virasoro primaries of the form (2.26) with scaling dimension one. From (2.24), it follows that the scaling dimension of V is

$$L_0 = \frac{1}{4}p_i^2 - \frac{1}{2}p^+(p^- + iQ) + N \,. \tag{2.27}$$

The non-standard contribution of p^+ to the scaling dimension is easy to understand. In string theory, the zero mode part of vertex operators for the emission of string modes have the general form

$$V = g_s \Psi \,, \tag{2.28}$$

where Ψ is the wavefunction of the state. Usually, the factor of g_s in (2.28) can be neglected since it is constant, but here according to (2.1) it is time-dependent and therefore needs to be retained. The vertex operator (2.26) corresponds to the wavefunction

$$\Psi(\vec{X}, X^+, X^-) = e^{i\vec{p}\cdot\vec{X} - ip^+X^- - i(p^- + iQ)X^+} . \tag{2.29}$$

Thus, the light-cone energy is

$$E^- = p^- + iQ \tag{2.30}$$

and (2.27) reads

$$L_0 = \frac{1}{4}\left(\vec{p}^2 - 2p^+E^-\right) + N . \tag{2.31}$$

The mass shell condition then becomes

$$m^2_{\text{eff}} \equiv 2p^+E^- - \vec{p}^2 = 4(N-1) . \tag{2.32}$$

The classical evolution of fields in this background is easy to describe. For example, consider a scalar field T with mass m in the light-like linear dilaton background. The lagrangian is proportional to

$$\mathcal{L} = \frac{1}{2}e^{2QX^+}(2\partial_+T\partial_-T - \partial_iT\partial_iT - m^2T^2) . \tag{2.33}$$

The equation of motion of T is

$$(2\partial_+\partial_- - \partial_i\partial_i + 2Q\partial_- + m^2)T = 0 . \tag{2.34}$$

A basis of solutions is given by

$$T(X^+, X^-, \vec{X}) = e^{-QX^+}e^{-ip^+X^- - iE^-X^+ + i\vec{k}\cdot\vec{X}} , \tag{2.35}$$

with

$$-2p^+E^- + \vec{p}^2 + m^2 = 0 . \tag{2.36}$$

2.3 Light-cone string field theory

The calculation of the perturbative string amplitudes of the light-like linear dilaton background becomes particularly simple in the light-cone gauge. Of course, given that the dilaton itself picks a preferred light-cone direction, one does not even break Lorentz invariance by making the usual gauge choice $X^+ = p^+\tau$ on the world-sheet. The world-sheet theory for the transverse coordinates is completely identical to that of flat space superstring theory.

As is well known from the old literature, one can represent a perturbative string amplitude in terms of a sum over light-cone diagrams. The contribution of each diagram is expressed as an integral over the positions τ_i of the joining and splitting operators on the world-sheet. For a genus g contribution to a n-string scattering amplitude the number of these vertex operators is $2g - 2 + n$. The effect of the linear dilaton is that the coupling constants now becomes a function of the light-cone coordinate τ on the world-sheet.

Specifically, every joining/splitting operator gets multiplied by $e^{Qp^+\tau_i}$. Hence the overall amplitude, before integrating over the τ_i, gets multiplied by

$$\prod_{i=1}^{2g-2+n} e^{-Qp^+\tau_i} \equiv e^{-(2g-2+n)Qp^+\tau_*} \,. \tag{2.37}$$

Here τ_* is the average of the insertion points τ_i. Because the world-sheet theory is translation invariant in τ, the rest of the integrand only depends on the relative differences of the positions τ_i of the joining/splitting vertices. This fact can be exploited by separating the integration over the τ_i into the integral over the relative positions multiplied by the integral over τ_*. The integral over the relative positions precisely gives the usual amplitude in flat space. We thus obtain the following simple relation between the string amplitudes in the light-like linear dilaton background and the corresponding flat space amplitudes:

$$A^{g,n} = A^{g,n}_{\text{flat}} \int_{-\infty}^{+\infty} d\tau_* e^{-(2g-2+n)Qp^+\tau_*} \,. \tag{2.38}$$

Clearly, the integral diverges even before summing over the genus. Hence, one clearly has to introduce a cut-off in the τ^* integral, keeping it away from $\tau = -\infty$. But even when we take $\tau_* > \tau_c$ one finds that the effective coupling $g_s^{\text{eff}} \sim e^{-Qp^+\tau_c}$ can become large when τ_c is negative. Therefore another description is required in this region. We will provide such a description in the next section.

3. Matrix string description

We will begin our discussion of Matrix theory by taking the flat space Matrix string action and inserting the time-dependent string coupling, $g_s = \exp(-QX^+)$. This leads directly to supersymmetric Yang-Mills on the Milne orbifold as the Matrix description of the light-like linear dilaton space-time. In section 3.1, we will provide an independent derivation leading to this same conclusion. This derivation will allow us to describe the regime of validity of the Matrix description.

Matrix string theory is described by a $(1+1)$-d super-Yang-Mills (SYM) theory with 16 supercharges. The action follows from dimensional reduction of $(9+1)$-d SYM theory. It contains eight matrix-valued fields X^i representing the transverse bosonic coordinates, as well as eight matrix-valued spinor coordinates Θ^a. The action is [17–19]

$$S = \frac{1}{2\pi\ell_s^2} \int \mathrm{Tr}\left(\frac{1}{2}(D_\mu X^i)^2 + \theta^T \not{D}\theta + g_s^2\ell_s^4\pi^2 F_{\mu\nu}^2 - \frac{1}{4\pi^2 g_s^2\ell_s^4}[X^i, X^j]^2 + \frac{1}{2\pi g_s\ell_s^2}\theta^T\gamma_i[X^i, \theta]\right) . \tag{3.1}$$

The metric on the world-sheet is flat, i.e. $\eta_{\mu\nu} = \mathrm{diag}(-1, 1)$, and the spatial coordinate σ on the world-sheet has a fixed periodicity equal to $2\pi\ell_s$. Notice that the Yang-Mills coupling constant, which is dimensionful in $(1+1)$ dimensions, is here identified with the inverse product of the string length and the string coupling,

$$g_{YM} \equiv \frac{1}{g_s\ell_s} \,. \tag{3.2}$$

In the IR limit the SYM theory become strongly coupled and, as shown in [19], reduces to the perturbative description of the type-IIA superstring. In the UV, however, the SYM theory is weakly coupled.

In the light-cone gauge the world-sheet time coordinate is proportional to the space-time null coordinate X^+. We can thus describe the light-like linear dilaton background in a simple way by allowing the string coupling to depend on the world-sheet time τ via a relation like $g_s = e^{-Q\tau}$. In section 3.1, we will determine the precise proportionality constant between X^+ and τ leading to (3.39).

It thus appears that we are dealing with a SYM theory with a time-dependent coupling constant. However, there is another way to view this result; namely, by changing the geometry on the world-sheet. Unlike the usual string action, the matrix string action is not conformally invariant: rescaling the metric by a function $f(\tau)^2$ changes the terms involving the coupling g_s in such a way that g_s gets multiplied by $f(\tau)^{-1}$. Thus, we conclude that the matrix string description of the light-like linear dilaton background is given by $(1+1)$-d SYM theory with fixed coupling, but on a world sheet with geometry

$$ds^2 = e^{2Q\tau}(-d\tau^2 + d\sigma^2)\,. \tag{3.3}$$

In fact, this metric is flat since it reduces to the usual Minkowski metric $ds^2 = -2d\xi^+ d\xi^-$ through the substitution

$$\xi^{\pm} = \frac{1}{\sqrt{2}Q} e^{Q(\tau\pm\sigma)}\,. \tag{3.4}$$

However, since the coordinate σ is periodic modulo $2\pi\ell_s$, the $(1+1)$-d Minkowski space described by the coordinates (ξ^+, ξ^-) turns into the Milne orbifold because of the identifications

$$\xi^{\pm} \equiv e^{\pm 2\pi Q\ell_s}\xi^{\pm}\,. \tag{3.5}$$

3.1 A more detailed derivation

We would like to extend the derivation of Matrix theory given in [20] (see also [21]) to our time-dependent example. We start with the ten-dimensional string metric

$$ds^2 = -2dX^+ dX^- + \sum_{i=1}^{8} (dX^i)^2 \tag{3.6}$$

and the light-like linear dilaton

$$\phi = -QX^+\,. \tag{3.7}$$

In discrete light-cone quantization (DLCQ), we make the identification

$$X^- \sim X^- + R \tag{3.8}$$

and focus on a sector with N units of light-cone momentum,

$$p^+ = \frac{2\pi N}{R}\,. \tag{3.9}$$

In [20], the theory with the identification (3.8) was defined as a limit of a space-like compactification, where the shift (3.8) of X^- is accompanied by a small shift of X^+. However, shifting X^+ is not a symmetry of our background (3.7), so we have to define the DLCQ in a different way. To that effect, we single out one direction X^1 from among the X^i and make the identification

$$(X^+, X^-, X^1) \sim (X^+, X^-, X^1) + (0, R, \epsilon R)\,, \tag{3.10}$$

where in the end we will take $\epsilon \to 0$. The Lorentz transformation

$$\begin{aligned} X^+ &= \epsilon x^+\,, \\ X^- &= \frac{x^+}{2\epsilon} + \frac{x^-}{\epsilon} + \frac{x^1}{\epsilon}\,, \\ X^1 &= x^+ + x^1 \end{aligned} \tag{3.11}$$

puts the background in the form

$$ds^2 = -2dx^+dx^- + \sum_{i=1}^{8}(dx^i)^2\,, \tag{3.12}$$

$$\phi = -Q\epsilon x^+\,, \tag{3.13}$$

with the identification

$$x^1 \sim x^1 + \epsilon R\,. \tag{3.14}$$

In this background, we focus on a sector with N units of momentum in the x^1 direction. After a T and an S duality, and introducing

$$r \equiv \frac{\epsilon R}{2\pi\ell_s}\,, \tag{3.15}$$

we are studying a sector with N D1-branes wrapped around x^1 in the type-IIB background

$$ds^2 = re^{\epsilon Q x^+}\left\{-2dx^+dx^- + \sum_{i=1}^{8}(dx^i)^2\right\}, \tag{3.16}$$

$$\phi = \epsilon Q x^+ + \log r\,, \tag{3.17}$$

with the identification

$$x^1 \sim x^1 + \frac{2\pi\ell_s}{r}\,. \tag{3.18}$$

This is now a theory of D1-branes in a background where the string coupling becomes weak near the big-bang, and strong at late times. It is worth stressing that this is opposite to the behavior of the string coupling in our original background (3.6), (3.7).

We now need to find a ground state of the D1-brane theory, and study fluctuations about this ground state. First consider a single D1-brane. If in the Dirac-Born-Infeld action,

$$S_{D1} = -\frac{1}{2\pi\ell_s^2}\int d\tau d\sigma e^{-\phi}\sqrt{-\det\left(\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} + 2\pi\ell_s^2 F_{\alpha\beta}\right)}\,, \tag{3.19}$$

we first put $F_{\alpha\beta}$ to zero, then the x^+ dependence cancels between the inverse string coupling and the determinant of the metric. So a simple classical solution is

$$x^1 = \frac{1}{r}\sigma\,, \tag{3.20}$$

$$x^+ = \frac{1}{r}\frac{\tau}{\sqrt{2}}\,, \tag{3.21}$$

$$x^- = \frac{1}{r}\frac{\tau}{\sqrt{2}}\,, \tag{3.22}$$

$$x^i = 0 \qquad i = 2, \ldots, 8\,. \tag{3.23}$$

If we now choose the gauge

$$\begin{aligned} x^1 &= \frac{1}{r}\sigma\,, \\ x^+ &= \frac{1}{r}\frac{\tau}{\sqrt{2}}\,, \end{aligned} \tag{3.24}$$

and define a new coordinate y by

$$x^- = \frac{1}{r}\frac{\tau}{\sqrt{2}} + \sqrt{2}y\,, \tag{3.25}$$

then, ignoring a total derivative, (3.19) can be expanded to give

$$\begin{aligned} S_{D1} = \frac{1}{2\pi\ell_s^2}\int d\tau d\sigma\Bigg(&-\frac{1}{r^2} + \frac{1}{2}\left[(\partial_\tau y)^2 + (\partial_\tau x^i)^2 - (\partial_\sigma y)^2 - (\partial_\sigma x^i)^2\right] \\ &+ 2\pi^2\ell_s^4 \exp\left(-\frac{\sqrt{2}\epsilon Q\tau}{r}\right) F_{\tau\sigma}^2 + \cdots\Bigg)\,, \end{aligned} \tag{3.26}$$

with

$$\sigma \sim \sigma + 2\pi\ell_s\,. \tag{3.27}$$

This agrees with the $N = 1$ case of (3.1) after rescaling the fields. Note that the coordinate y appears on the same footing as the x^i $(i = 2, \ldots, 8)$, so it plays the role that x^i used to play before we made the compactification space-like.

For N D1-branes, the world-volume theory is given by (3.1).

3.2 Regime of validity of the Matrix string description

The modes (2.35) of a scalar field in the lightlike linear dilaton background (3.6), (3.7) take the following form in terms of the new coordinates (3.11):

$$T(x^+, x^-, \vec{x}) = e^{-\epsilon Q x^+}\exp\left\{-i\left(\epsilon E^- + \frac{p^+}{2\epsilon} - k_1\right)x^+ - i\frac{p^+}{\epsilon}x^- + i\left(k_1 - \frac{p^+}{\epsilon}\right)x^1 + i\sum_{j=2}^{8} k_j x^j\right\}. \tag{3.28}$$

The identification (3.10), or equivalently (3.14), implies the momentum quantization condition

$$p^+ = \epsilon k_1 - \frac{2\pi n}{R}\,. \tag{3.29}$$

In DLCQ, we focus on a sector with given N and study fluctuations that stay within that sector. Such fluctuations have $n = 0$, so that

$$p^+ = \epsilon k_1 \,. \tag{3.30}$$

Using (3.30), the mass shell condition (2.36) for non-negative m^2 implies

$$|k_1| \leq 2\epsilon |E^-| \,. \tag{3.31}$$

As a consequence, (3.28) shows that the energy and momentum in the new coordinate system (x^+, x^-, x^i) are at most of order

$$\epsilon E^- \,. \tag{3.32}$$

Taking into account the identifications (3.24), we conclude that the world-sheet energy and momentum appearing in the action (3.26) are at most of order

$$E_{typical} \sim \frac{\epsilon E^-}{r} \sim \frac{E^- \ell_s}{R} \,. \tag{3.33}$$

The effective time-dependent string length $\ell_s^{\rm eff}$ can be read from the metric (3.16),

$$\ell_s^{\rm eff} = \frac{\ell_s e^{-\epsilon Q x^+/2}}{\sqrt{r}} \,. \tag{3.34}$$

The condition for open string oscillators to decouple is given by

$$\epsilon E^- \ell_s^{\rm eff} = \sqrt{\frac{2\pi\epsilon \ell_s^3}{R}} E^- e^{-\epsilon Q x^+/2} \ll 1 \,, \tag{3.35}$$

which is satisfied in the $\epsilon \to 0$ limit.

We must also check that gravity decouples from the Matrix description. The effective ten-dimensional Newton "constant" can be determined from the metric (3.16) and dilaton (3.17),

$$G_N^{\rm eff} \sim g_s^2 (\ell_s^{\rm eff})^8 = \frac{\ell_s^8 e^{-2\epsilon Q x^+}}{r^2} = \frac{4\pi^2 \ell_s^{10} e^{-2\epsilon Q x^+}}{\epsilon^2 R^2} \,. \tag{3.36}$$

Taking into account the fact that the energies of fluctuations are given by (3.32), we see that the fluctuations interact gravitationally with strength

$$G_N (\epsilon E^-)^8 \sim \frac{4\pi^2 \ell_s^{10} \epsilon^6 (E^-)^8 e^{-2\epsilon Q x^+}}{R^2} \,, \tag{3.37}$$

so closed strings also decouple for $\epsilon \to 0$.

So complete decoupling of closed and massive open strings can be achieved by strictly setting

$$\epsilon = 0 \,. \tag{3.38}$$

This strict limit makes perfect sense from the SYM point of view, since its coupling

$$g_{YM} = \frac{1}{g_s \ell_s} = \frac{1}{\ell_s} \exp\left(\frac{\sqrt{2}\pi \ell_s Q \tau}{R}\right) \tag{3.39}$$

and the typical energies (3.33) are ϵ-independent. In other words, by using the limit (3.38), we reach the remarkable conclusion that our Matrix description is valid all the way to the singularity, which opens up the perspective of using it to study the fate of the singularity.

Now that we have argued that the Matrix description is a complete description of the physics of the singularity, we should ask whether the description is weakly coupled. The relevant dimensionless parameter is the ratio of the SYM coupling to the typical energy of processes we are interested in. Using (3.39), (3.33), (3.40) and

$$p^+ = \frac{2\pi N}{R}, \tag{3.40}$$

we see that this parameter equals

$$\frac{g_{YM}}{E_{typical}} \sim \frac{N \exp\left(\frac{\sqrt{2}\pi \ell_s Q \tau}{R}\right)}{p^+ E^- \ell_s^2}. \tag{3.41}$$

Thus for any fixed finite N the Matrix description is strongly coupled for late times and weakly coupled for early times. Note, however, that in DLCQ, we eventually want to take the decompactification limit $N \to \infty$, $R \to \infty$ with fixed $p^+ = 2\pi N/R$. This is corresponds to the limit where we take $N \to \infty$ holding g_{YM}^2 fixed while considering energies of order $1/N$. In a strict $N \to \infty$ limit, which undoes the DLCQ by decompactifying the lightlike circle, one finds a strong coupling behavior for all times. One could also imagine having N depend on time and keeping an appropriate combination of the parameter (3.41) and N almost fixed close to the singularity, thus effectively decompactifying the DLCQ circle near the singularity while keeping the SYM theory weakly coupled near the singularity. For some range of times, it might also be useful to perform an analysis along the lines of [22].

3.3 Cosmological evolution and the emergence of space-time

Time evolution from the big bang to late times corresponds beautifully to renormalization group flow in the Yang-Mills theory. At the big bang, the Yang-Mills theory is weakly coupled. Since the coupling has positive mass dimension, weak coupling corresponds to the UV sector of the theory. As time evolves, the coupling increases and the Yang-Mills theory flows to the IR.

At early times, we have weakly coupled Yang-Mills theory. Since the coupling (3.2) is small, the potential terms in (3.1) turn off as we approach the big bang. We are left with a theory of non-commuting matrices. These appear to be the correct degrees of freedom near the singularity, replacing our conventional notion of space-time. At very late times, we recover light-cone quantized perturbative string field theory in the light-like linear dilaton background, along the lines described in [19].

In Matrix theory descriptions of flat space-time, supersymmetry plays a critical role. Roughly speaking, diagonal matrix elements are interpreted as positions in space-time (they are the positions of the D1-branes in their transverse space), while off-diagonal matrix elements correspond to strings connecting the various D1-branes. When two well-separated clusters of D1-branes are considered, for instance corresponding to two well-separated supergravitons, the off-diagonal modes between the two clusters are heavy and can be integrated out, a priori giving rise to an effective potential for the separation modulus of the two clusters. Supersymmetric cancellations ensure that the effective potential vanishes and that the moduli space metric is flat [23]. This is crucial for the space-time interpretation of Matrix theory: if supergravitons interacted with a static rather than velocity-dependent potential, the model would not describe gravity in flat space-time.

Indeed, a Matrix theory description of a non-supersymmetric flat string background with a closed string tachyon was given in [24]. This Matrix theory develops a potential that lifts the flat directions, a pathology that was given the interpretation that the original non-supersymmetric space-time is not a solution of non-perturbative string theory.

Our model, the light-like linear dilaton background of type-IIA string theory, preserves 16 supersymmetries, so one might have hoped to find a supersymmetric Matrix theory description. However, it turns out that supersymmetry is spontaneously broken in any sector with non-zero light-cone momentum $p^+ = 2\pi N/R$. Since in our discrete light-cone quantization we focus on a sector with a fixed non-zero value for N, we are bound to find a Matrix string description in which supersymmetry is broken. This would have been disastrous if the breaking were explicit.

However, what we actually found is maximally supersymmetric two-dimensional Yang-Mills theory in flat space with a boost identification. The boost identification breaks all the supersymmetry but only via boundary conditions on the Milne circle: namely, the action of the boost transformation acts differently on fields of different spin leading to a different quantization condition for bosons and fermions [10]. This is an effect that becomes less relevant as the Milne circle grows in time. At late times, the potential becomes small as supersymmetry is restored. It would be interesting to understand this potential in a quantitative way.

3.4 Does time begin?

The deepest question that we would hope to address with this formalism is whether the big bang should be thought of as the beginning of time, or whether space-time exists prior to the singularity. From a space-time point of view, it is not clear that it makes sense to continue the spacetime metric (2.13) beyond the singularity at $\lambda = 0$.

The same question has been addressed for the Milne orbifold as (two dimensions of) a space-time background in perturbative string theory [5, 6]. The conclusion was that $2 \to 2$ scattering amplitudes across the singularity diverge at tree-level because of large tree-level gravitational backreaction from the region close to the singularity [6].

Our Matrix description a priori contains only the future quadrant of the Milne orbifold as the Matrix string world-sheet. One could try to include the other three quadrants of the Milne orbifold and see whether states can be propagated across the singularity. Although there is now no gravitational backreaction, one can show along the lines of [6] that large gauge backreaction gives rise to UV enhanced IR divergences similar to the ones found in [3]. These divergences are in some sense milder than those encountered in the gravitational case, but they might still be problematic in our low dimension field theory. However, it is not entirely clear that these divergences are associated with the singularity.

It is also possible that the correct prescription involves selecting an initial condition at the big bang and considering only the future quadrant of the Milne orbifold. There are two natural states to choose as an initial vacuum: the conformal vacuum annihilated by positive frequency modes with respect to the conformal time η given in (1.5), and the adiabatic vacuum inherited from the underlying Minkowski space. For a more detailed discussion, see [25, 6]. At the big bang, we have a conformal field theory since the Yang-Mills theory is free. The natural vacuum from the perspective of conformal field theory would be the conformal vacuum which is SL(2) invariant. However, the adiabatic vacuum has the advantage of better high-energy behavior at the singularity, and is the vacuum state that is usually used in string theory computations [8, 6]. At late times, where the perturbative string description is good, it is natural to study states with reference to conformal time which is identified with X^+ in space-time via (3.24).

It is an interesting question to determine the precise observables in this Matrix model. For example, one possibility could be to take a natural initial Yang-Mills state and ask how it evolves into a collection of excited strings in the space-time that emerges at late times.

4. Some generalizations

4.1 A dual type-IIA background

There are many interesting ways to generalize the light-like linear dilaton solution. For example, given the M-theory metric (2.4), we can compactify the X^9 direction and interpret it as the M-theory circle. This gives rise to an alternate type-IIA description, with metric

$$ds_{10}^2 = e^{QX^+}[-(dX^0)^2 + (dX^1)^2 + \cdots + (dX^8)^2] + e^{-QX^+}(dY)^2 \tag{4.1}$$

and light-like linear dilaton

$$\phi = \frac{QX^+}{2}\,. \tag{4.2}$$

In a coordinate basis, the metric (4.1) has non-vanishing curvature components

$$\begin{aligned} R_{+i+i} &= \frac{Q^2}{4}e^{QX^+}\,, \\ R_{+y+y} &= -\frac{3Q^2}{4}e^{-QX^+}\,. \end{aligned} \tag{4.3}$$

Thus we find that the Ricci tensor has a non-vanishing component

$$R_{++} = Q^2\,. \tag{4.4}$$

The non-vanishing Ricci tensor is supported by the dilaton, which in the presence of the metric (4.1) makes a non-vanishing contribution to Einstein's equations:

$$\nabla_+\nabla_+\phi = -\Gamma^+_{++}\partial_+\phi = -\frac{Q^2}{2}\,. \tag{4.5}$$

Like (2.4), the space-time (4.1) is singular as $X^+ \to -\infty$ because the metric components $\tilde{g}_{ii}$ go to zero in that limit.

If we now compactify the X^8-direction, $X^8 \sim X^8 + 2\pi R^8$, and T-dualize, we find a type-IIB solution with metric

$$ds^2_{10} = e^{QX^+}[-(dX^0)^2 + (dX^1)^2 + \cdots + (dX^7)^2] + e^{-QX^+}[(dX^8)^2 + (dY)^2] \tag{4.6}$$

and constant dilaton,

$$\phi = \log\left(\frac{\ell_s}{R^8}\right) \tag{4.7}$$

with $X^8 \sim X^8 + 2\pi\ell_s^2/R^8$.

4.2 The light-like linear dilaton in type-IIB

We could also directly consider the light-like linear dilaton in type-IIB string theory. The new ingredient in type-IIB that we should consider as $g_s \to \infty$ is S-duality. The Einstein frame metric (1.4) is invariant under S-duality, but we obtain a new string frame metric valid in the strong coupling regime. We can view the resulting S-dual description as either string theory with a time-independent string length ℓ_s but with a coupling and metric

$$\widetilde{g_s} = e^{QX^+}\,, \qquad ds^2 = e^{QX^+}\left\{-2dX^+dX^- + \sum_i (dX^i)^2\right\}, \tag{4.8}$$

or as string theory with a coupling and metric

$$\widetilde{g_s} = e^{QX^+}\,, \qquad ds^2 = \left\{-2dX^+dX^- + \sum_i (dX^i)^2\right\}, \tag{4.9}$$

and a time-dependent string length $\widetilde{\ell_s}^2 = e^{-QX^+}\ell_s^2$. As we approach the singularity at $X^+ \to -\infty$, the effective string tension goes to zero. Once again, we see that the resolution of the singularity requires physics beyond the $\widetilde{\ell_s}$ expansion.

It might appear that because the string coupling is small near the singularity, we should have a good perturbative string description. If this were the case, we might hope to resolve the cosmological singularity in string perturbation theory. However, this is not the case: graviton perturbation theory, which is controlled by the (duality invariant) effective Newton constant, still breaks down near the singularity. We therefore expect a new description involving new degrees of freedom in the strong coupling regime. Such a description should follow from type-IIB Matrix string theory [26, 18].

4.3 The light-like linear dilaton from little string theory

Finally, it is worth mentioning that the light-like linear dilaton can be obtained as an unusual Penrose limit of the near horizon geometry of type-II NS5-branes studied in [27].[2] This geometry is described by

$$ds^2 = N\ell_s^2\left[-d\tilde{t}^2 + \frac{dr^2}{r^2} + d\Omega_3^2\right] + dy_5^2\,, \qquad e^{2\Phi} = \frac{N\ell_s^2 g_s^2}{r^2}\,. \tag{4.10}$$

The conventional time coordinate t has been rescaled, $t = \sqrt{N}\ell_s\tilde{t}$, to uniformize the factors of N appearing in the metric. There is also an NS three-form field strength H_3 which is N times the volume form of the three-sphere, but this will vanish when we take the limit.

We are interested in boosting along a radial rather than angular null geodesic in this space. To do this we switch coordinates $(\tilde{t}, r) \to (u, v)$ by $\tilde{t} = u - v$, $r = \sqrt{N}\ell_s e^u$. This gives

$$ds^2 = N\ell_s^2\left[2dudv - dv^2 + d\theta^2 + \cos^2\theta d\psi^2 + \sin^2\theta d\phi^2\right] + dy_5^2, \quad e^{2\Phi} = g_s^2 e^{-2u}\,. \tag{4.11}$$

Finally, to take the limit [27], we rescale $v \to v/N$, $\theta \to \theta/\sqrt{N}$, $\psi \to \psi/\sqrt{N}$, and take $N \to \infty$. This sends H_3 to zero since

$$H_3 \sim N \sin\theta d\theta d\psi d\phi\,, \tag{4.12}$$

which goes like $N^{-1/2}$. We are left with the metric,

$$ds^2 = \ell_s^2\left[2dudv + dx_3^2\right] + dy_5^2\,, \qquad \Phi = \Phi_0 - u\,, \tag{4.13}$$

which describes the light-like linear dilaton in flat space-time.

If we were now to study perturbative states in light-cone string theory on this background, we would like to understand to which states these correspond in the original coordinates. After the limit, the light cone energy and momentum are given by

$$\begin{aligned} 2p^- &= -i\frac{\partial}{\partial u} = -i\left(\frac{\partial}{\partial \tilde{t}} + r\frac{\partial}{\partial r}\right) = -i\left(\ell_s\sqrt{N}\frac{\partial}{\partial t} + r\frac{\partial}{\partial r}\right) \\ 2p^+ &= -\frac{i}{N}\frac{\partial}{\partial v} = \frac{i}{N}\frac{\partial}{\partial \tilde{t}} = \frac{i}{\sqrt{N}}\ell_s\frac{\partial}{\partial t}\,. \end{aligned} \tag{4.14}$$

The states of interest to us are those with finite p^- and p^+. Mapping these states back to the original variables, we see that these states comprise a certain sector of high-energy states in little string theory [28] with energies of order $\sqrt{N}$.

The main problem with this approach for obtaining a holographic description of the null dilaton is that little is still known about little string theory beyond its bulk definition. However, many other geometries give rise to the light-like linear dilaton via similar limits, and perhaps one of those geometries will provide a more tractable holographic dual in the spirit of the AdS/CFT correspondence.

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